We discussed divisibility and remainders many weeks ago. Today, we will use those concepts and discuss another type of question – successive division. But before we do, you need to go through the previous related posts on division if you haven't read them already:

Divisibility Unraveled

Divisibility Applied on the GMAT

Divisibility Applied to Remainders

Now, let's start working on today's topic.

What is meant by – 'a number when divided successively by 4 and 5 leaves remainder 1 and 4 respectively'?

Does it mean that when you divide the number by 4, the remainder is 1 and when you divide it by 5 the remainder is 4? No. It means that when you divide the number by 4, the remainder is 1 and then when you divide the quotient obtained (from the first division) by 5, the remainder is 4.

e.g., when you divide 37 by 4, the remainder you get is 1 and the quotient you get is 9. When you divide 9 (not 37 here) by 5, the remainder you get is 4.

This is what we mean by successive division i.e. you keep dividing quotients you get instead of starting with the original number again. Now that we understand what successive division is, let's look at what kind of questions appear on successive division.

Question 1: A number when divided successively by 4 and 5 leaves remainders 1 and 4 respectively. What will be the remainder when this number is divided by 20?

(A) 0
(B) 3
(C) 4
(D) 9
(E) 17

Solution: Let's find one number, say n, which when divided successively by 4 and 5 leaves remainder 1 and 4 respectively. When you divide n by 4, you get a quotient and remainder 1. When you divide the quotient by 5, you get the remainder 4. What can the quotient be (when you divide by 5) for the remainder to be 4? The first one that comes to mind is that the quotient could be 4 itself. If you divide 4 by 5, the remainder is 4. Now, if in the previous step when you divided n by 4, if the quotient was 4 and the remainder was 1, then the number n must have been 4*4 + 1 = 17 (n = Quotient*Divisor + Remainder). Now, what will be the remainder when you divide 17 by 20? It will be 17.

Answer (E)

Basically, you multiplied the second remainder with the first divisor and added the first remainder to get the first such number.

Two divisors: 4, 5

Two remainders: 1, 4

Start from the bottom right corner and go up diagonally: 4*4.

Then go down and add 1: 4*4 + 1

The first such number is 17.

If you want to see the algebra approach, let us show you that too.

$n = 4a + 1$ (When n is divided by 4, quotient is ‘a’)

$a = 5b + 4$ (When a is divided by 5, quotient is ‘b’)

$n = 4(5b + 4) + 1 = 20b + 17$

When n is divided by 20, we see that the remainder will be 17.

The problem with the algebra approach is that it gets a little cumbersome when dealing with 3 or more successive divisions. Let’s look at a question involving 3 divisions now.

Question 2: On dividing a certain number by 5, 7 and 8 successively, the remainders obtained are 2, 3 and 4 respectively. When the order of division is reversed and the number is successively divided by 8, 7 and 5, the respective remainders will be:

(A) 3, 3, 2
(B) 3, 4, 2
(C) 5, 4, 3
(D) 5, 5, 2
(E) 6, 4, 3

Solution:

Three given divisors: 5, 7, 8
Three given remainders: 2, 3, 4
We start by considering the last step first. At the end, when we divide by 8, we want remainder to be 4. This means that after division by 7, we should get 4 as the quotient (to get the first value of n). When we divide by 7, the quotient must be 4 and the remainder must be 3. At this step, the number becomes 7*4 + 3 = 31

Now, when we divide by 5, we need the quotient to be 31 and remainder to be 2 so number must be 5*31 + 2 = 157

157 is the first such number. Divide 157 by 8, 7 and 5, in that order:
157/8 gives quotient = 19 and remainder = 5
19/7 gives quotient = 2 and remainder = 5
2/5 gives quotient = 0 and remainder = 2

Remainders are 5, 5, 2

Answer (D)

The steps followed are these:

Three Divisors: 5, 7, 8
Three Remain: 2, 3, 4

Start from the bottom of the last column i.e. from the third remainder:

Go up diagonally and multiply by the second divisor: 4*7 = 28

Go down and add the second remainder: 28 + 3 = 31

Go up diagonally and multiply by the first divisor: 31* 5 = 155

Go down and add the first remainder: 155 + 2 = 157

157 is the first such number. Now proceed as before to get the remainders when you divide 157 by 8, 7 and 5.

Now you can tackle any number of divisors and remainders easily!